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***Savings Lost:  
The Damage of Taxable Advance Refundings to Taxpayers***

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**Appendix**

The key analytical concepts in this paper are savings (in present value terms) and option value. We will provide below a high-level description of the analytical approach to quantify the par optionless yield curve needed for the counterfactual analyses.

The key concept of valuing certain cashflows, including optionless bonds, is discounting. The discount rates can be stated explicitly but can also be obtained from a par optionless yield curve (or from the one-period forward rates). The discount rates can be converted to specify an optionless (par) yield curve. The key point is that the values of optionless bonds are the same, no matter how the discount rates are specified.

The analysis of the embedded call option requires additional input. The option's value depends on uncertain future interest rates, which must be modeled. The industry-standard (Bloomberg) approach is the so-called Black-Karasinski process, which is the one used in our paper. In addition to the current optionless yield curve, the B-K process is specified by two parameters needed for the interest rate process: the volatility of the short-term interest rate and a mean reversion factor. If the mean reversion factor is set to zero, as in our analysis, the B-K process reduces to the lognormal distribution. The volatility is estimated from the market prices of callable bonds rather than the historical movements of interest rates. The process is calibrated so that the values of optionless bonds are unaffected, i.e., they are the same as under conventional discounting.

The above procedure is called the Option Adjusted Spread (OAS) approach. Given the inputs (optionless yield curve, interest rate volatility), it can determine the fair value of a callable bond. Or, given the price of a bond, it can determine the required yield curve shift (OAS) that would result in the specified price. Details are provided in Chapter 36 by Fabozzi (2021). A textbook reference is Tuckman (2022), Part Three (Term Structure Models).

The above approach applies to all bonds, whether taxable or tax-exempt. In the case of taxable bonds, an issuer's yield curve is specified by spreads to the Treasury curve. These spreads can be estimated from the current prices of the issuer's optionless bonds. However, in the case of tax-exempt bonds, an optionless benchmark par yield curve is not readily available because the standard municipal yield curve is derived from the prices of 5% bonds, which are callable at par in year ten. Given the non-existence of optionless 15-year or 30-year optionless tax-exempt bonds, how can we estimate long optionless par yields?

The essential input to the estimation of optionless rates is the issuer's callable yields, specifically the YTC's based on the prices of the 5% NC-10 bonds. These yields are commonly specified by spreads to a AAA callable 5% NC-10 benchmark, such as the one provided by MMA. The optionless rates can be obtained from the callable rates by 'stripping out the call options.'

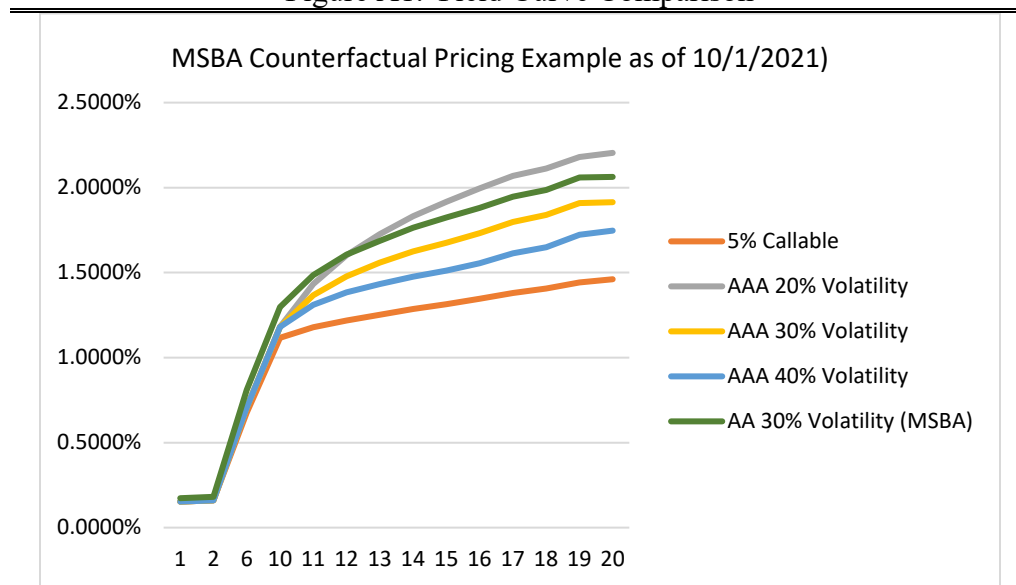
Yield curve stripping is performed at a specified interest rate volatility, and the selection of the volatility will be discussed below. Given the volatility, the procedure is as follows. The yields of optionless 5% bonds for the first ten years are available from the 5% NC-10 yield curve, and we can convert these into par yields, i.e., into a par yield curve for the initial ten years. Next, we solve for the 11-year optionless par rate, considering that the price of the 11-year NC-10 bond is known. Given an 11-year optionless rate, the OAS engine described above determines the price of the 11-year NC-10 bond. By numerical search, we can solve for the 11-year optionless par rate that validates the price of the 11-year NC-10 bond. With the same procedure, we iteratively determine the par rates for longer maturities and obtain an optionless par yield curve. See Kalotay (2017) for more details.

Table A1. Yield Curve Comparison, MSBA Counterfactual Pricing as of October 1, 2021

Maturity Date	Year	5% Callable	AAA 20% Volatility	AAA 30% Volatility	AAA 40% Volatility	AA 30% Volatility*
10/15/2022	1	0.1530%	0.1530%	0.1530%	0.1530%	0.1730%
10/15/2023	2	0.1610%	0.1612%	0.1612%	0.1612%	0.1810%
10/15/2027	6	0.6760%	0.7054%	0.7054%	0.7054%	0.8090%
10/15/2031	10	1.1170%	1.1803%	1.1803%	1.1803%	1.2980%
10/15/2032	11	1.1780%	1.4329%	1.3668%	1.3113%	1.4850%
10/15/2033	12	1.2190%	1.6025%	1.4786%	1.3834%	1.6060%
10/15/2034	13	1.2530%	1.7257%	1.5594%	1.4326%	1.6870%
10/15/2035	14	1.2860%	1.8311%	1.6254%	1.4766%	1.7640%
10/15/2036	15	1.3150%	1.9157%	1.6755%	1.5111%	1.8240%
10/15/2037	16	1.3470%	1.9960%	1.7330%	1.5561%	1.8810%
10/15/2038	17	1.3810%	2.0684%	1.7987%	1.6135%	1.9470%
10/15/2039	18	1.4060%	2.1132%	1.8388%	1.6489%	1.9870%
10/15/2040	19	1.4420%	2.1801%	1.9098%	1.7228%	2.0590%
10/15/2041	20	1.4610%	2.2043%	1.9142%	1.7470%	2.0630%

\* MSBA Example

Figure A1. Yield Curve Comparison



The final hurdle is the choice of interest rate volatility, which refers to the volatility of short-term interest rates. The guiding light is the ratio of the optionless tax-exempt rate to the identical maturity Treasury rate. The higher the volatility, the lower the estimated tax-exempt rate, and the lower this ratio will be. We note that the ratio is observable for the first ten years, because municipal bonds shorter than 11 years are not callable. The ratios must be realistic – for example, according to previous studies, they should increase with maturity, and for investment-grade issuers, the ratio cannot exceed 1 (Kalotay & Dorigan, 2008). Under normal conditions, the ratio of short-term tax-exempt rates to short-term Treasuries depends on individual investors' maximum marginal tax rate. During the period of interest, the highest marginal tax rate was roughly 40%; therefore, the short-term tax-exempt rate was expected to be 60% of the like-maturity Treasury rate. Previous empirical studies of tax-exempt bonds issued near par

demonstrate that the ratio of the tax-exempt rates to the taxable rates depends on the prevailing and expected tax rates (Poterba, 1986). This ratio increases with maturity and may reach 90% at the 30-year point. Based on the actual ratios of tax-exempt to taxable rates between 2019 and 2021, we found that a 30% interest rate volatility provides reasonable estimates of the optionless rates.

Table A1 and Figure A1 detail various yield curves as of October 1, 2021, the counterfactual pricing date of the MSBA refunding. The AAA par optionless yield curves are at different volatilities (20%, 30%, and 40%). As a reminder and discussed in the paper, we adjusted the AAA par optionless rates to AA par optionless rates for MSBA since the issuer was rated AA at the time of the counterfactual refunding.